Mathematics

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An Outline for Mathematics' Classrooms and Teaching

Erasmus+ program "QuaMMELOT"

Prologue: Core considerations

The main purpose of what follows is to consolidate skills of "curricular elaboration" for every Teacher who teaches Mathematics; these elaborations are targeted – and therefore validated as such – towards the Teachers' each-time student-population. To that, skills and techniques of formative assessment (and corresponding classroom-material) are required.

A side-purpose is **to embed didactical techniques and views to the set of didactical tools already at hand** for Teachers who teach Mathematics, so as to facilitate their ability and willingness to assess and elaborate differentiated approaches to their own teaching methods and material.

A special purpose is to upgrade the consciousness that **"multiple representations"** (inherent in whatever conceptualization of Mathematics as an episteme one accepts) provide for

- didactical conceptualizations, along with
- posing "problems" in the general consideration of the term, and
- open approaches to teaching (as, e.g. approaches of "theory born via the teaching-action itself") that support differentiated engagements with learning and teaching.

Along with the above, we aim at adding value to teaching via the adoption of a widening of the actively-learning community,

- either via "cooperative learning" and "peer-tutoring" (the latter would/could preferably facilitate inclusiveness),
- or by introducing thematics / problems / tasks of an "open nature" (such as cultural or social thematics could give rise to), in order to engage a wider audience (as, e.g. families or/and close surroundings) in the 'learning activity; that would support our effort towards schooling'.

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Our indicative list of proposed tasks – a bigger and more elaborated set of which will constitute the crucial component of the web-seminar – aspire to serve, at least, the above targets.

It is crucial to point out that we do not think of our list simply as a set of "take-and-do" classroom-tasks, but rather as a set of initial ideas that would – and should – become the starting point of corresponding (co-) elaboration of all those engaged in the web-seminar, preferably via "a net of interactions from a distance" that we hope to thus help create.

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Introductory remarks

What follows is an attempt to describe a framework within which the educative task, corresponding to teaching secondary Mathematics in a classroom whose students' population is multi-cultural and with varying educational loads.

The content abides (as is expected, in order for it to be applicable in classrooms as is) to given contexts, as described e.g. by the corresponding math-curricula; It tends, however, towards the formulation of a "set of general principles of teaching" applicable – as such – in various educational environments and settings, as those corresponding to the varying semantics described in the math-curricula and teaching-traditions of the countries engaged in the Erasmus+ program "QuaMMELOT".

The material exposed here is meant to facilitate and support rather the personal reflections of teachers engaged, than the use of this material as is. This is the content and context of most of the "Working material", exposed as voluntary tasks, after each paragraph.

The following aims are considered as crucial, therefore are apt to criticism and assessment within the outlined framework by the teaching-practitioners in secondary schools:

1. **Consolidating skills of "didactical elaboration" of curricular contents** profoundly taking into account the students'-population these are targeted towards.
2. **Providing "didactical techniques"** that should be added to those already at hand (or be considered and reflected upon, if already at hand) for teaching practitioners, in order to facilitate their rationale of the each-time didactical approach adopted for the referred classes.
3. **Outlying and establishing the added value of calling upon the** – inherent in Mathematics in their historical evolution, therefore due to be put to use – **"multiple representations" and "open methods-approach"** for the shaping of one's teaching material and aims; these, at the same time, are expected to support differentiation in teaching, engagement and learning.
4. **Broadening the practitioners' conceptualizations as concerns their teaching program, towards "group-work" and "collaborative/peer learning", also contextualized as "open-problems"**; we foresee the-

se as crucial attitudes for the facilitation of schooling, on the one hand, and inclusion for pupils with a refugee background, on the other hand, not to forget they might also support "poor-learners" or/and "fast-learners" in the fulfillment of their subjective needs and demands, the more since "open-problems" (problèmes ouverts) could serve as the points of intersection of the classroom and "home", in the sense that they could be elaborated as a sequence of overlapping tasks-and-results in the classroom and "at home", e.g. as in the case of problems that refer to social instances or/and every-day issues.

5. **Exhibiting indicative examples of such didactical scenarios** (as opposed to "plans of teaching", that tend to refer to certain aims of a didactical session – limiting themselves to it – and are validated according as to the fulfillment of these aims, and not in accordance to their didactical impact); these scenarios are meant to critically reflect upon, and to not as "take-and-do".

Furthermore, one should also consider the need for certain institutional/state initiatives that would provide for the appropriate framework to facilitate the given task. These do not belong to our task here, but the pilot-implementation in classrooms of the ideas and attitudes elaborated as part of the Erasmus+ project "QuaMMELOT", should give rise to corresponding reflections as part of the professional learning of teachers engaged, so we briefly turn to corresponding ideas.

Working material

i. Reflect upon Your expectations as concerns points 1. – 5. Communicate Your corresponding thoughts, if You wish, via the forum.

An outline of institutional measures under consideration

Given the need for an as swift as possible realization of the aim of inclusion in the every-day life and tasks of schools for our pupils with a refugee-background, we acknowledge the need for an institutional frame to be provided by the state/the state-institutions responsible.

Firstly, **these pupils need to find the space and time for their "preparation"** for the novice demands they are faced with, as concerns not only schooling (a hard task for an amount of them, given their long-term

absence of any such activity or institutionalization), but also the need to attend classes that are being taught in a more-or-less unknown language to them.

Specific courses, offered preferably by or in the schools they are going to attend, and as a part of their time at school, would/could/should, at least, consider specific aims, as the following:

6. The Introductory Assessment and establishment of attitudes towards and conceptualization of schooling as such, preferably conceived of as a secure learning and self-fulfilling environment with certain prerequisites, as e.g. the adoption of certain rules and forms of behavior.
7. The Introductory, but also Formative Assessment of the pupils' pre-existing or missing readiness to deal with tasks provided as "working papers", including the "symbolic texts" that are crucial for mathematics tasks.
8. In conjunction to the latter, it should also be assessed in what extend pupils are apt to express themselves in the framework of a mathematics-classroom (Mathematical Literacy or "Matheracy") with the use of symbolic language or/and via the formulation of a corresponding poster-presentation, for example when exhibiting their proposed solutions or elaborations of a mathematics-task; the poster-presentation would/could serve as a first step (if the pupils are already accustomed to such tools) towards their engagement with mathematics-tasks as members of the whole pupils' team in the inclusive classroom.

Considering the above, we think of **a "parallel introductory program"** that should serve as a first step towards the adoption of attitudes considered as preferable for the facilitation of inclusion, and should cover some – not all – of the teaching-hours for the referred pupils.

One of the initiatives that we could consider as preferable would be the co-teaching (Cooperative Teaching) of a language- and of a mathematics-teacher, so as to facilitate the learning of (introductory?) language-forms along with a framing of it (that would provide for corresponding conceptualizations) within the problem-solving and symbolic classroom-framework of the mathematics-classroom.

The Greek team is referring to already existing experience, such as the Bilingual Schools of the Hellenic Republic in Bavaria (Bayern, F.R. Germany), and to experience gained via the corresponding pilot-

implementation of Literacies in Secondary Technical Schools, where texts of corresponding technical material (taught in corresponding classes) were elaborated towards the conceptualization and appointment of meaning of mathematical concepts.

Furthermore, and specifically, the co-teaching experience, gained through the corresponding pilot-implementation of it in secondary technical schools in Greece in the period 2017-2022, funded by European funds, implies that this "classroom-architecture" supports and facilitates the engagement of pupils in general, the more if one considers the pupils of a refugee background and their specific characteristics and needs towards inclusion. Let it be noted that a number of incoming-pupils of this group are already attending these classes in the school-year 2018-2019.

This implementation aims concretely at facilitating the Differentiated Pedagogy and the differentiated approach to teaching and learning, indicatively via:

9. the personalized approach of the pupils,
10. the support of active "working groups", preferably structured as to the role of each member,
11. emulation of working teams, e.g. in problem-solving tasks,
12. opportunities for questions – at times, recurrent problems of knowledge-gaps – to be answered without any side-effects,

but also

13. the role of teachers in supporting the inclusive/cooperative/supportive classroom-environment.

Working material

ii. Make a list of institutional measures already implemented by Your state, as concerns the facilitation of the work undertaken by You and Your math-co-teachers in order to support schooling and inclusion for pupils of a refugee-background. Communicate Your material, if You wish, via the forum.

iii. Would You propose further institutional measures, or locally-implemented ones, apart from those listed above? Communicate Your corresponding thoughts, if You wish, via the forum.

On the specific characteristics of the proposal outlined here

What is exhibited here aims at supporting and facilitating the *active inclusion* of pupils of a refugee-background; a significant particle of such an approach is the establishment and functional implementation of team-work (working in groups during the elaboration of tasks) and/or peer-tutoring (as an introductory attitude towards inclusion).

It is, in fact, considered here that these novice classroom-attitudes (if at all novice) would support the inclusion and active engagement of the whole pupils' population, in that they support, promote and facilitate differentiation throughout the classroom-activity (considered as either subjective and personalized, or structuring the teaching-learning interaction as a whole).

Concrete methodological characteristics that are conceived of also as aims are:

- Empowering pupils as such, through their own recollections of experience, as well as through supporting their inquiries; to that, we consider the potential of the engagement of their close surroundings (be it their family or else) as rather preferable.
- Supporting the acknowledging of either common grounds or differences – in their historical (world-heritage) or/and cultural background, the least as an initiation of corresponding positive reflection towards inclusiveness.
- Producing a repository of didactical material (as, e.g. resources, practices, interesting and effective thematics that would serve as "enriching modules" of the each-time curricular content, classroom reports and good practices, structural and leadership proposals), apt to be called upon by all and each participating country, after their weighing via implementation; these could be used freely also by other countries or/and educators.
- Supporting, via corresponding notions and tools, teachers to understand and elaborate the multiple and varying identities and potentials of their pupils (e.g. as concerns their cognitive loads, their cultural-cognitive heritages and the corresponding activity choices).
- Engaging the whole school-communities in reflections upon the added value and possibly the challenges arising from the social- and learning-inclusion of the pupils of a refugee-background, within a bottom-up and, at the same, common European environment.
- Supporting the learning and adoption of the language of the European country via its implementation in *problem-solving* and *group-*

work: Given the methods described here (such as using posters to exhibit and communicate – at times iconically – their contribution to the classroom-activity), pupils are expected to *gradually* acknowledge and incorporate the value itself of using both the symbolic, as well as the language of the European country of their current residency.

The above characteristics, also the need to weigh and co-operate in sharing material, add value to the multi-national character of "QuaMMELOT", at the same time indicating towards comparative elaborations and contextualizations.

In this sense, what is referred to above for the Greek educational system needs to provoke corresponding references from the other participating members (the latter is asked for in the corresponding "Working material").

Corresponding methodological proposals

Differentiated teaching for Inclusion

The existing experience calls for the adoption and materialization of Differentiating approaches to teaching, that take into account the differentiated attitudes towards learning of our pupils.

We don't mean here – we do not exclude this interpretation, though – to personalized approaches to teaching, as e.g. in the case of pupils that are supported exclusively by expert teachers when diagnosed with certain learning difficulties: even if it may be needed at times, sticking to that model means the exclusion of the pupil, instead of our sought for inclusion.

We don't, furthermore, claim it is possible to concretely differentiate our pupils as to their "cognitive styles", so as to rationalize our approach to each and every one of them.

What is referred to here as "Differentiated approach to teaching and learning for inclusion" tends towards and implies *the formulation of teaching-modules or/and teaching scenarios* that abide to the following characteristics:

14. Presenting the material to be elaborated in the classroom in accordance to the demand of "multiple views" of the same material, so as to facilitate varying approaches.

15. In conjunction also to the previous, be apt to accept and build upon "alternative presentations" of both the tasks (see below), and the solutions proposed by our pupils (methodologically divergent thoughts, but also poster- and depictive presentations).
16. Formulating "peer-working-teams", where roles are to be appointed successively, but characteristics stemming out of our Formative Assessments should also (at least, to begin with) be taken into account.
17. Even if "peer-working-teams" are not a choice, the accordingly formulated teaching-modules provide for a frame of active engagement of more pupils, *if we see to it that the time-schedules foresee time for the composition of varying views and proposals and for the timely and step-by-step establishment of "what is to be learned/conceptualized", leaving aside the common attitude of "providing a given* (standardized) *and correct view" from the beginning*.

To each problem arising within the thus described frame, one could call upon didactic methods of a "collaborative classroom" that do not merely describe the "peer-working-teams" as such, but facilitate *post-cognitive skills* to be put to use, as, for example:

18. Pupils that are already apt to deal with the "formal" content of the taught module are expected (fulfilling the demands of differentiation that call upon "conclusive and interesting tasks for each according to his/her special demands) to exhibit *conclusions towards the elaboration of "open problems" in the spirit of inquiry, that is establishing the "formal" conclusion via experimentation of verifications and falsifications*.

Tools presented here should facilitate the formulation of appropriate modules.

Working material

iv. Consider a mathematics-module that is being taught in secondary classes in Your country and, after discussing the posed aims of the teaching, make a list of tasks that would support differentiated learning within the curricular frame that the above aims form. Communicate Your outcome, if You wish, via the forum.
v. Consider a mathematical notion that is part of the curriculum of secondary classes and describe (at least) two different approaches for the formulation of a corresponding school-book

text or teaching-module. Communicate Your material, if You wish, via the forum.