From arithmetic to algebraic expressions

A(n) (educational) scenario about teaching 15-year-old-students

in a regular class including 'newly arrived' students

This scenario is planned for teaching and learning Mathematics[1] in a regular class, including 'newly arrived' students (refugees or immigrants whose mother tongue is one of following: Arabic, Farsi, Kurdish, Turkish, Urdu), of a mainstream school[2],

where they are taught:

- in a special class: the host country's language as a foreign one for 15 teaching hours per week
- in a regular class: other subjects, Mathematics included, of the host country curriculum for 20 teaching hours per week.

A. Before ……. The 'Language' Class

The teacher who teaches Mathematics, in cooperation with the teacher who teaches Language in a teaching hour of the special class, using a special dictionary, teach the 'newly arrived' students the basic words needed for the forthcoming Mathematics lesson.
During this teaching hour of the special class the two teachers co-teach. Ideally their classmates from the 'regular' class participate in this language lesson.

a. Construction of my self-constructed dictionary

they provide their students the table below and with the help of the mathematical dictionary they already have, they are asked to complete the table with the corresponding word in their mother tongue. They asked them to work cooperatively in groups of 3-4. They We suggest every group to consist of students who speak different languages, for giving them a chance to exchange their experiences.

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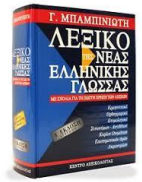
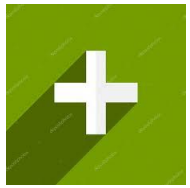
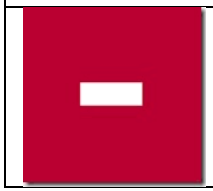
	**Greek:** **(lexiko)** **English:** **Arabic:** **Urdu:** **Turkish:** **Kurdish:** **Farsi:**	**ΛΕΞΙΚΟ** **λεξικό** **dictionary**
	**Greek:** **(arithmos)** **English:** **Arabic:** **Urdu:** **Turkish:** **Kurdish:** **Farsi:**	**ΑΡΙΘΜΟΣ** **Αριθμός** **number**
	**Greek:** **(sin)** **English:** **Arabic:** **Urdu:** **Turkish:** **Kurdish:** **Farsi:**	**ΣΥΝ** **συν-πρόσθεσε** **plus-add**
	**Greek:** **(plin)** **English:** **Arabic:**	**ΠΛΗΝ** **Πλην-αφαίρεσε** **minus-substract**

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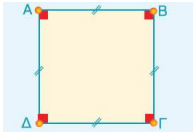
	Urdu: **Turkish:** **Kurdish:** **Farsi:**	
	**Greek:** **(epi)** **English:** **Arabic:** **Urdu:** **Turkish:** **Kurdish:** **Farsi:**	**ΕΠΙ** **Επί-πολλαπλασίασε** **times-multiply**
	**Greek:** **(dia)** **English:** **Arabic:** **Urdu:** **Turkish:** **Kurdish:** **Farsi:**	**ΔΙΑ** **Διά-διαίρεσε** **by -divide**
	**Greek:** **(tetragono)** **English:** **Arabic:** **Urdu:** **Turkish:** **Kurdish:** **Farsi:**	**ΤΕΤΡΑΓΩΝΟ** **τετράγωνο** **square**

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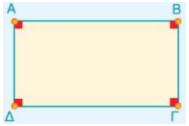
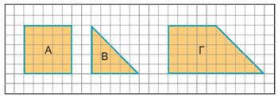
	**Greek:** **(orthogonio)** **English:** **Arabic:** **Urdu:** **Turkish:** **Kurdish:** **Farsi:**	**ΟΡΘΓΩΝΙΟ** **ορθογώνιο** **rectangle**
	**Greek:** **(emvadon)** **English:** **Arabic:** **Urdu:** **Turkish:** **Kurdish:** **Farsi:**	**ΕΜΒΑΔΟΝ** **εμβαδόν** **area**

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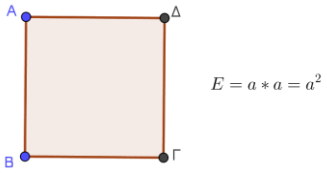
b. Mathematical Dictionary

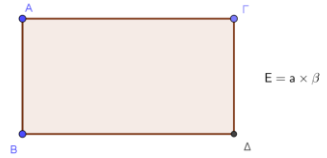
As far as the students complete their dictionary above, the two teachers try to shape opinion whether their students have any idea of these geometrical objects and if not, they help them to find out the 'obvious properties' with examples.

1. Square/The area of a square



$$E = a * a = a^2$$

2. Rectangle/The area of a rectangle



$$E = a \times \beta$$

'Objectives for newly come students'

After completing this lesson the student should be able

- to have their one part of dictionary completed
- to recognise the corresponding Greek words
- to distinguish the dimensions of a square and a rectangle and hopefully be able to calculate their area.
- To communicate their relevant experiences from their home country educational system.

B. The 'Mathematics' class

The two teacher, after the special class, talk about how they are to work during the Mathematics lesson. Drawing conclusions from the way their students respond to this first attempt they plan their lesson.

From arithmetic to algebraic expressions

Time: Two teaching hours

Objectives for all students

After completing the lesson the students should be able to:

- know about identities in algebraic expressions and be able to represent them geometrically.
- grasp and elaborate to an extent the relationship between arithmetic, algebra and geometry
- generalise and transfer their learning over toother topics and contexts
- draw links between representations and develop new mental images for concepts.

The classroom orchestration

The students work collaboratively in small groups of 3-4 students.

Material required

- A set of cards, containing also blank cards
- Markers
- A poster, scissors, glue
- Calculator

Previous knowledge

The students should be familiar with

- basic symbols like +,-,* (times), /(divided by) ,=
- the corresponding basic operations
- calculating the area of a square and a rectangle

Implementation

The two teachers work collaboratively and ask their students to work in groups of their choice. They provide them with a set of cards as below, containing blank ones. They ask them tocut and stick their cards down onto a poster in rows. Every row should consist of a representation of the 'same' algebraic expression (verbal, geometric, table, equalities). If there cannot find a card they can use their marker to complete a blank one with the missing piece. Dictionaries and calculators spare for students' use.

<table>
<tr>
<td>$3\alpha * a$</td>
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<td>Πολλαπλασίασε με 3 το τετράγωνο του α</td>
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<tr><td>α</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
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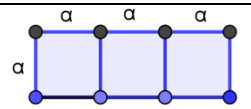
<td> 
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<td>$3\bullet\alpha^2$</td>
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<td>πολλαπλασίασε το 3 με α και ύψωσέ το στο τετράγωνο
ή
Ύψωσε το τριπλάσιο του α στο τετράγωνο</td>
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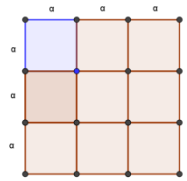
<td>  </td>
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2. Completed Poster (the answer)

<table>
<tr>
<td>$(3\alpha)^2$</td>
<td>πολλαπλασίασε το 3 με α και ύψωσέ το στο τετράγωνο
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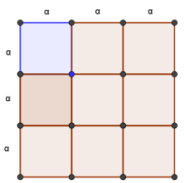
<td>  </td>
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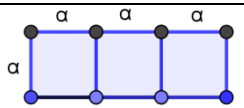
<td> 
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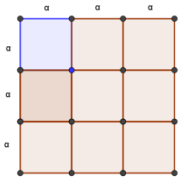
<td>$9\bullet\alpha^2$</td>
<td>Πολλαπλασίασε με 9 το τετράγωνο του α</td>
<td><table><tr><td>α</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>απάντησε</td><td>9</td><td>36</td><td>81</td><td>144</td></tr></table></td>
<td>  </td>
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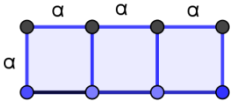
<td> 
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After having finished, they stick the cards on their posters, they write down on them the names of the members of the group and the work carried out by each of them and finally, they put the posters on the wall and compare their answers. They try to find out the similarities and discuss the differences.

Added value

- Participation, Inclusion and empathy
- Build on previous knowledge
- Build a partly self-constructed list to refer to
- Content and language integrated learning
- Active learning
- Cooperative learning and teaching
- Differentiated learning

Theoretical framework of the scenario

The above scenario is created, based on relevant research, adapted to the circumstances under which the Greek teachers are obligated to teach Mathematics. It is an example of a classroom task aspiring to help teachers create their own. From our point of view there is no theory for producing 'take-to-do' educational material, even if wanted, due to the level of diversity in the Greek classroom.

Our starting point was the idea of giving all students, no matter if they came to stay or leave, education based on principles of racial and cultural equality and a commitment for full participation within a democratic society. Cummins, after 25 years of research on bilingual education, he demanded (Cummins, 2000) for a 'transformative/intercultural pedagogy' for language-minority students where students' language and cognitive abilities are engaged in the learning process and where students' identities are affirmed', which also has to include the needs of the rest students. We took into account the issues raised in relation to the essential role Mathematics play, not only in the Greek Curriculum and school society, but all over the world. (D'Ambrosio, 2007, 2015) sees mathematics as the dorsal spine of civilization, the basis of

science and technology. Consequently, parents use to spend a lot of money for tutoring and hold high expectations for their children's achievement in Mathematics, mostly unrealistic. Despite Mathematics teachers' efforts, students, especially in their later school years, are not able to meet the demands of curriculum. So Mathematics proves to be a discipline for a few, raising an issue of social justice (Ernest 2007).

'Newly arrived students' bring with them an additional load for the teachers, who already struggle to cope with these bias and often are accused that 'hold very stereotypical views about their students' ability to learn mathematics, and thus offer a dumped down curriculum to their students' (Matthews et al. 2005).

So we started to plan an educational scenario of a real time class in Greece, having in mind that it should be applicable for all students. All students should see it as worth doing, human, quirky and interesting, if possible, by the use of multiple representations (Wood et al., 2007).

We had to take into consideration the perception of the objectives of mathematical education and teaching mathematics of the Greek Curriculum of Mathematics relevant to multiple representations (FEK 162/22-01-2015, page 12*)*:

'To be familiar with an effective use of the language and tools of Mathematics

Representations of concepts, processes and relations.

This ability includes the understanding of the ways different kinds of representations in Mathematics are used and connected (spoken language, symbols, images, diagrams, tables), the recognition of the advantages and constrictions and the flexibility of choice the proper representation for the demands of the problem-to be solved. In Upper High School we usually talk about arithmetic, geometric, graph, algebraic and verbal representations or for software representations. It is essential for the students to be familiar with the different representations, each of which will also illuminate different aspects of the mathematical concept or process.

Then we had to look for tools used in other relevant conditions. We drew from the research for intercultural and multilingual education, inclusive education (for special needs students) and general Mathematics education. Mainly four tools proved to be similar and very helpful to our point of view:

1. Content Language Integrated Learning (known as CLIL), which is supposed to be an umbrella term for immersion, content based and bilingual education programs in America, Europe and Asia. Navés T. (2009) comes to the conclusion that efficient CLIL programs share ten common characteristics; we tried to be satisfied under the given circumstances:

(1) respect and support for the learner's first language and culture;

(2) competent bilingual teachers, that is, teachers fully proficient in the language of instruction and familiar with one of the learners' home languages;

(3) mainstream (not pull-out) optional courses;

(4) long-term, stable programmers and teaching staff;

(5) parents' support for the programs

(6) cooperation and leadership of educational authorities, administrators and teachers;

(7) dually qualified teachers (in content and language);

(8) high teaching expectations and standards;

(9) availability of quality CLIL teaching materials and

(10) properly implemented CLIL methodology.

Our scenario tried to meet these characteristics:

(1) Due to high diversity there is no chance, for the time being, to have bilingual teachers for 'newly arrived students'. But in this task students are asked to communicate their mother tongue by writing it down and exchange with the rest class.

(2), (7) we suggest the co-teaching of Language and Mathematics teacher.

(3) (8) It is a part of the curriculum and they work all together

(6) there is an attempt to face it

(4), (5), (9), (10) are under question

2. Differentiated Teaching and Learning

Both of our classes, the special and the regular ones are mixed classes of any kind, as every class in smaller extent possibly. The last 30 years has been developed the idea of Differentiated Classroom, the "Artful teaching", which is regarded as a learning triangle (the teacher, the kids, and the 'stuff') with the teacher being atop (Tomlinson, 1999). We propose such a teacher, who does not carry out recipes, but feels free to edit the list, to revise it, to add and subtract from it as he sees fit.
The Teacher, who always has in mind the principles of differentiating teaching in any time, especially while teaching and
(1) appreciates each Child as an Individual;

(2) remembers to teach whole Children;

(3) continues to develop expertise;

(4) strives for joyful learning;

(5) offers high expectations - and lots of ladders;

(6) shares the teaching with Students ;

(7) helps Students make their own Sense of ideas;

(8) high teaching expectations and standards;

(9) clearly Strives for Student Independence;

(10) thinks "Discipline" is more covert than overt;

(11) uses positive energy and humour.

3. Cooperative Learning

A teacher of the kind we described before, in a differentiated classroom, takes care of the other point of the triangle, the student. Swan M., who aspired us for the task, points out (Swan 2006) that collaborative learning methods promote robust, transferrable learning that endures over time or that may be used in non-routine situations. The rationale is building on the knowledge students already have, exposing and discussing common misconceptions, creating connections between topics. Additionally, our students by working in groups exchange their experiences, share their difficulties, learn from each other and communicate their findings with the whole class. By this peer learning seems to be implemented Cummins's 'transformative/intercultural pedagogy' we invoked before.

4. Co-teaching

We suggest co-teaching, first met as a paradigm in inclusive education, as a way (paraphrasing Cook and Friend' Rationale for Co-Teaching, 2004):

1. to deliver services to our 'newly come students' as part of a philosophy of inclusive practices in our mixed class

2. to give ALL students the chance to receive improved instruction. This includes students who are academically gifted or talented, students who have average ability, students who are at risk for school failure, students with identified special needs, as well as 'newly come students'.

It is also very important for our students working collaboratively to have their teachers as a pattern of a society who helps each other and interact for better results. An active community who accepts each other and motivate each other to engage in discussing and explaining ideas, challenging and teaching one another, creating and solving each other's questions and working collaboratively to share methods and results.

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