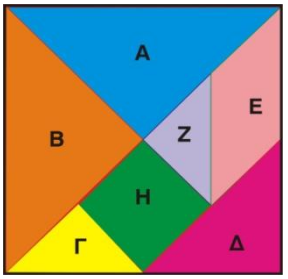
Scenario Puzzle

- **The main idea**
  Through game driven activities, we will discuss the notion of fractions in parallel to the notion of area, by using transformations (shift, turn, symmetry), addition and comparison of fractions, multiples.
- **History**
  - Tangram, Chinese game.

The **tangram** (Chinese: 七巧板; pinyin: *qīqiǎobǎn*; literally: "seven boards of skill") is a dissection puzzle consisting of seven flat shapes, called *tans*, which are put together to form shapes. The objective of the puzzle is to form a specific shape (given only an outline or silhouette) using all seven pieces, which may not overlap. It is reputed to have been invented in China during the Song Dynasty and then carried over to Europe by trading ships in the early 19th century. It became very popular in Europe and then again during World War I. It is one of the most popular puzzles in the world.



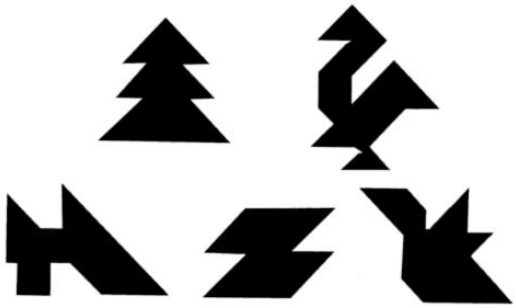
Co-funded by the Erasmus+ Programme of the European Union

Choosing a unit of measurement so that the seven pieces can be assembled to form a square of side one unit and having area one square unit, the seven pieces are

- 2 large right triangles (hypotenuse 1, sides √2/2, area 1/4)
- 1 medium right triangle (hypotenuse √2/2, sides 1/2, area 1/8)
- 2 small right triangles (hypotenuse 1/2, sides √2/4, area 1/16)
- 1 square (sides √2/4, area 1/8)
- 1 parallelogram (sides of 1/2 and √2/4, area 1/8)

Out of these seven pieces, there is only one parallelogram as it has no reflection symmetry but only rotational symmetry, and so its mirror image can be obtained only by flipping it over. Thus, it is the only piece that may need to be flipped when forming certain shapes.

Shapes:

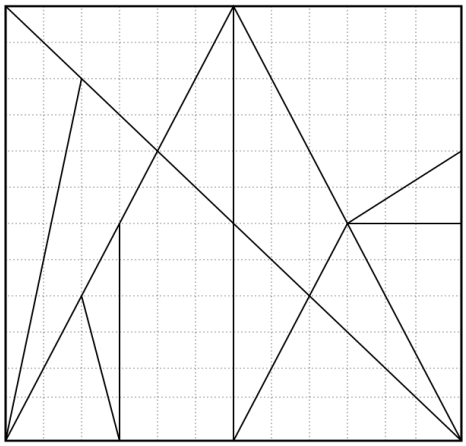


- Ostomachion, ancient Greek game

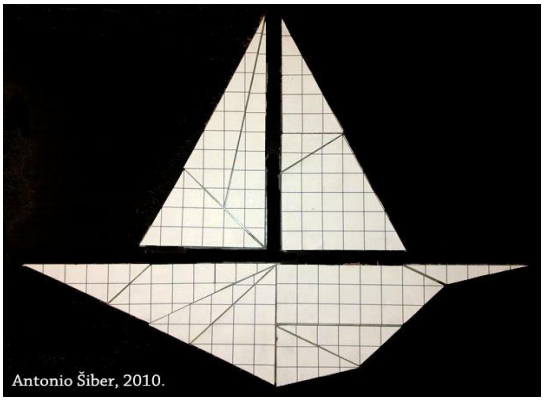
Ostomachion, also known as ***loculus Archimedius*** (Archimedes' box in Latin) is a mathematical treatise attributed to Archimedes. This work has survived fragmentarily in an Arabic version and a copy, the Archimedes Palimpsest, of the original ancient Greek text made in Byzantine times. The word Ostomachion originates from the greek word Ὀστομάχιον, which means "bone-fight", from ὀστέον (*osteon*), "bone" and μάχη (*mache*), "fight, battle, combat". Note that the manuscripts refer to the word as "**Stomachion**", an apparent corruption of the original Greek. The Ostomachion, which he describes, was a puzzle similar to tangrams and was played perhaps by several persons with pieces made of bone.

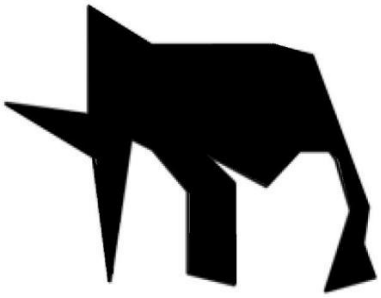
It is a… game, ancestor of puzzles. It consists of a rectangular base divided in 14 geometric pieces. The aim of the game is to reform with as many as possible the rectangle or one of the 9 predefined shapes (a helmet, a flying hen, a tower, a column, an elephant, a boar, a barking dog and a stalking hunter).
One important result shown by Archimedes is that the area of the rectangular base is an integer multiple of the area of each one of the 14 geometric pieces.

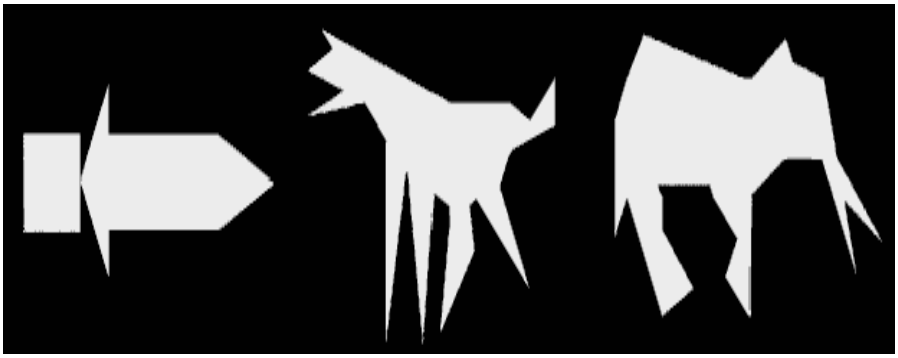
Co-funded by the Erasmus+ Programme of the European Union



Shapes:







- **Objectives:**

-To mobilize all of the students, including those that have a difficulty expressing themselves in the language of the class, through the usage of work sheets without words.

-With the participation of all the students of the class, to enrich the mathematical vocabulary of the students with terms like rectangle, triangle, area, fraction, multiples and others.

-In the teaching and learning progress: a) Multiple representations (from the algebraic to the geometric and vice versa as well as the representation through letters).b)the notion of fraction as a part of a whole but the whole in respect to the parts as well (using integer numbers).

- **Description of activities**

The worksheets that we propose are, didactically speaking, as open as they can possibly be. Addressing a "mixed" class (in which we have refugees participants) we try to provide multiple representations with very little to no words. The notation A, B, AB, ΓHZE… refer to additions of areas (in the Euclidean way) or additions of fractions. Notations like the following:

 ---------- 4A

refer to multiples...

1. In the work sheet 1, the students will use a rectangular piece of carton with dimensions 10x10 cm, geometric drawing tools and scissors. They will draw the lines corresponding to the Tangram and they will cut the 7 pieces in order to play with them. The correct drawing of the shapes is a first, elementary mathematical activity.
2. In the work sheet 2, the students, who can also work in pairs, can use whichever method they chose. The possible methods are: the observation of the shape by comparing different areas (geometric), the addition of the fractions for the computations of the second table (algebraic), the use of geometric transformations (turn, slide, symmetry), and other. In that part, the notion of fraction is connected to the notion of integer multiple (the relationship between the part and the whole and vice versa).
3. In the work sheet 3, the students, working as they did for the work sheet 1, will draw and cut the 14 pieces of the Ostomachion. Then they will try to piece together at least one of the shapes provided.

4. In the work sheet 4, the students will try to piece together their own images using the 14 pieces they already have and they will suggest the shape they create as a puzzle to the other student (they will keep their solution a secret).
An optional extension of the above can be for the student to try and calculate the fractions or the multiples of the part in respect to the whole. For example, in the triangle 7 of the filo ergasias is the 1/12 of the initial rectangle.

- **Added value**

-These activities can be done by students of various levels and languages (we do not forget that we are working on "mixed" classes).

-Getting accustomed with shapes on the plane, informal usage of mathematical transformations on the plane.

-The understanding of the notion of the fraction as part of a whole and vice versa.

-The connection between the algebraic and the geometric frame, in the Euclidean sense, broadens the mathematical horizons of the students and leads them towards a more global and intuitive perception of mathematics.

-The development of logical reasoning, in the form of "if that, then the other".

-Boost in creativity by shaping their own images.

-Development of a cooperating culture between students.

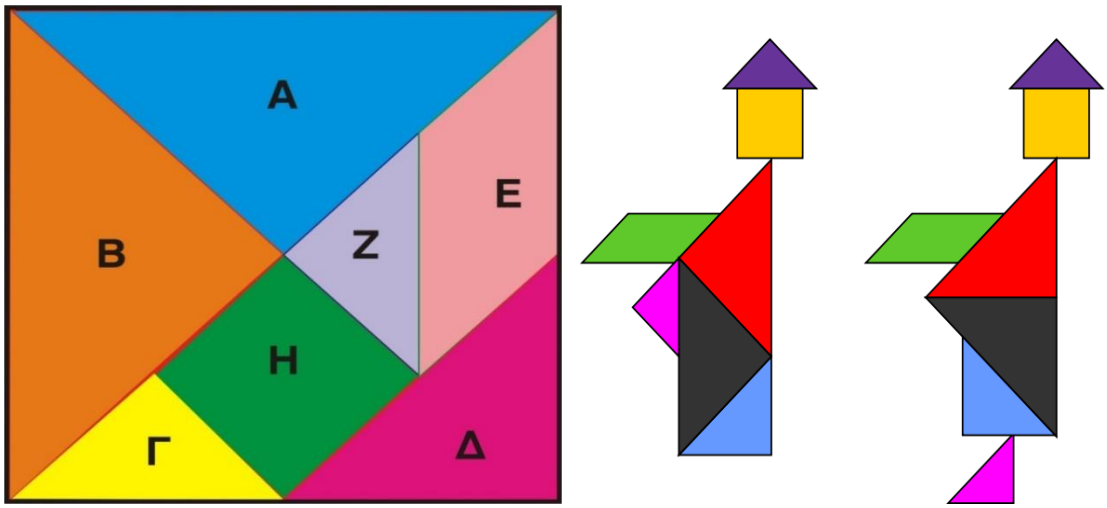
- **Observations:**

-The proposed scenario can, with appropriate modifications, be addressed to classes of Elementary school, higher classes of High school (for example the equality between the triangles of the Ostomachion) as well as higher levels of education, if we take in mind that Archimedes was concerning himself with problems of combinatorics.

-The historical references, for example the life and work of Archimedes can be a subject for the students to do creative projects on.

work sheet 1

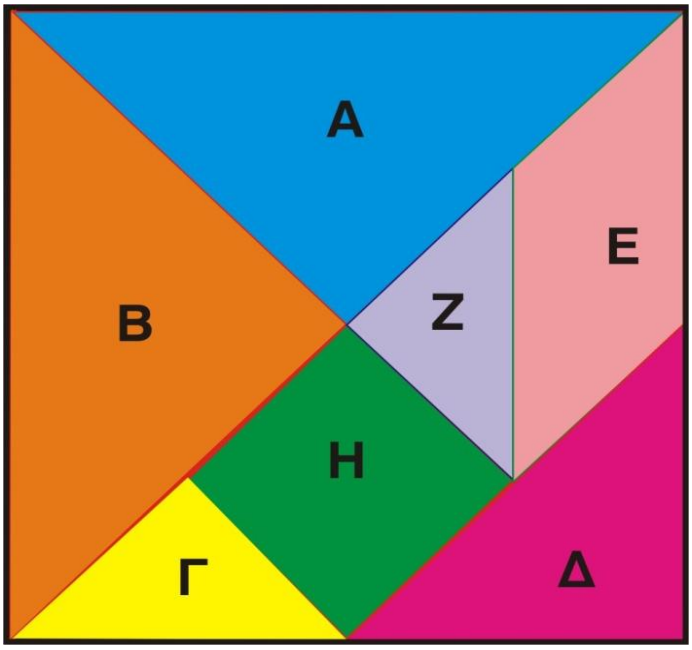
tangram



Let's play

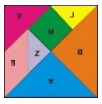
Co-funded by the Erasmus+ Programme of the European Union

work sheet 2

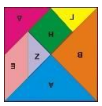


4A---------- 

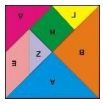
16Z --------- 

…B---------- 

…Γ---------- 

…Δ ---------- 

…H---------- 

…E --------- 

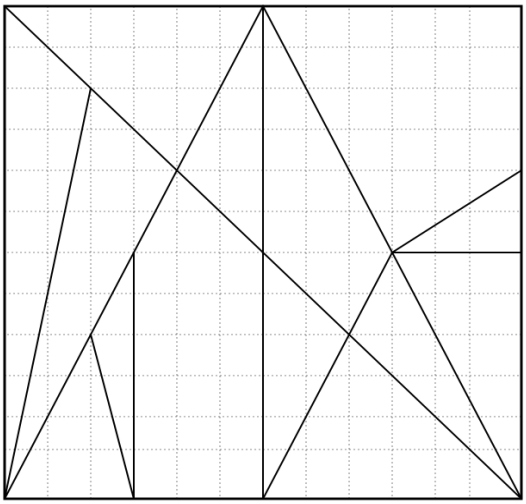
A	B	Γ	Δ	E	Z	H
1/4			1/8			

AB	EΔ	ΓZ	ΔH	ΓΔ	ΓHZE
	1/4				

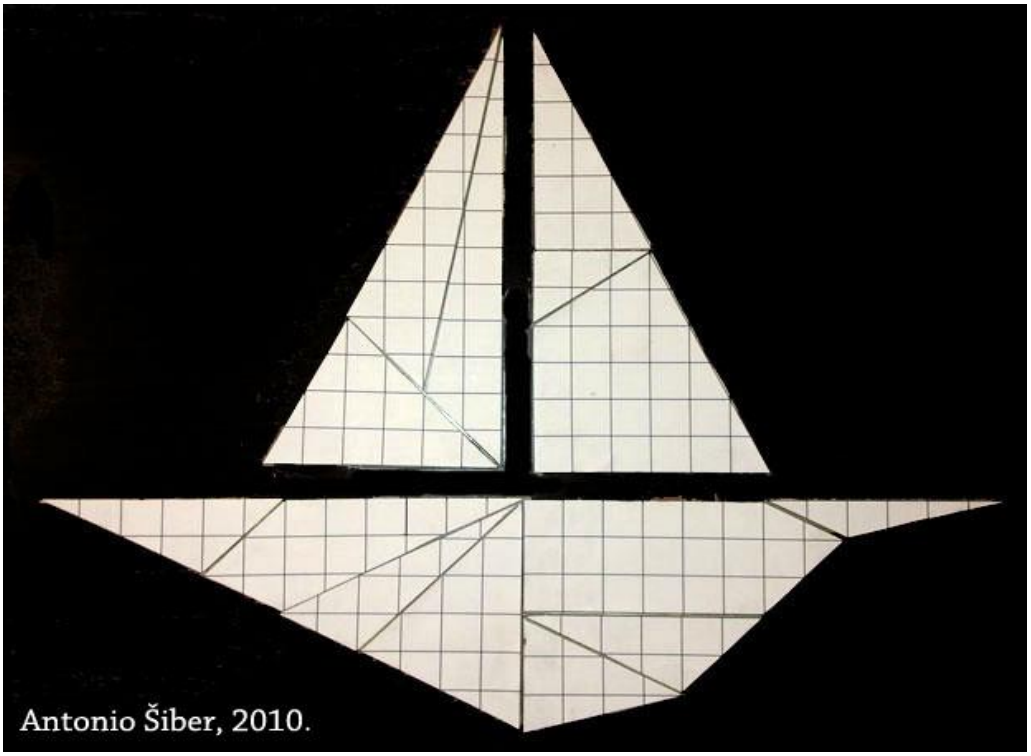
Co-funded by the Erasmus+ Programme of the European Union

work sheet 3

Ostomachion

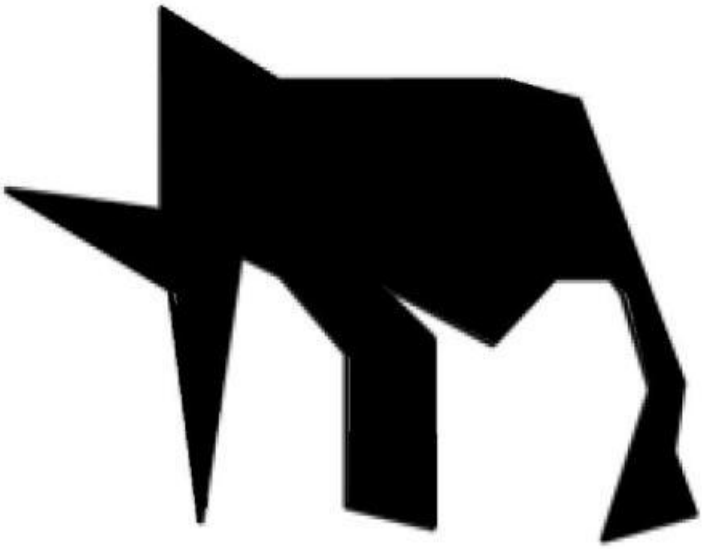


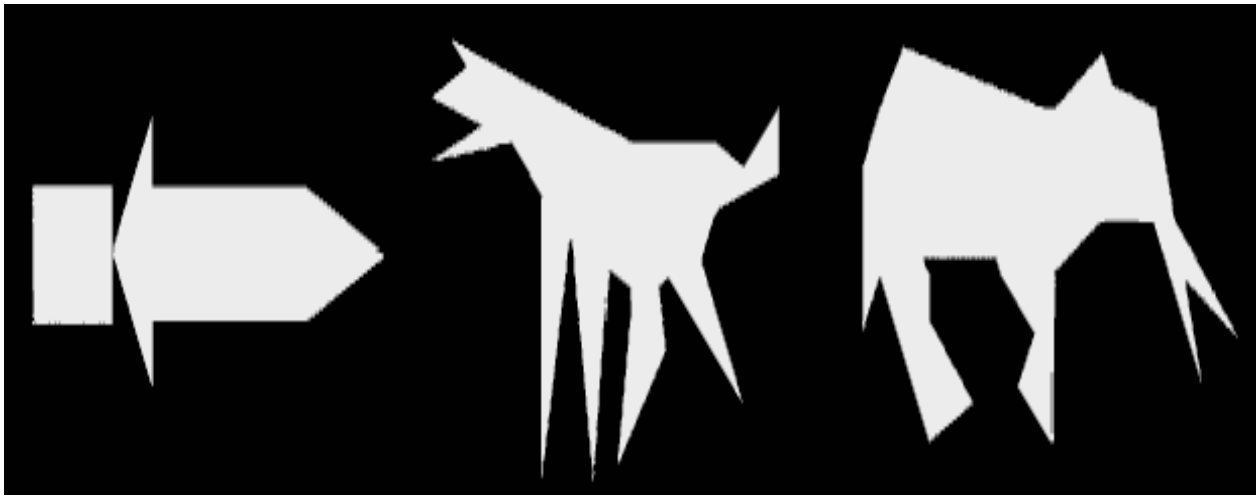
Image



Co-funded by the Erasmus+ Programme of the European Union

Let's play

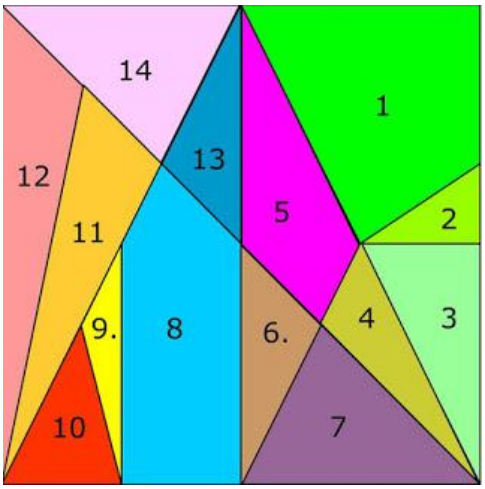


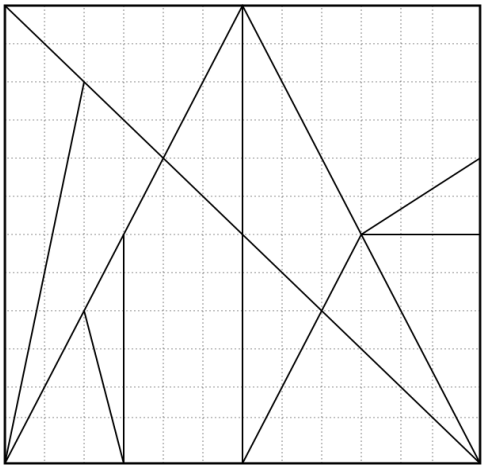


Co-funded by the Erasmus+ Programme of the European Union

work sheet 4

Piece together your own image.





Co-funded by the Erasmus+ Programme of the European Union

References

Reviel Netz, Fabio Acerbi, Nigel Wilson (2004), Towards a Reconstruction of Archimedes' *Stomachion*, SCIAMVS 5 67-99 *Stanford University Venzone (un), Italy Lincoln College, Oxford*

J. L. Heiberg, (1881), Archimedis opera omnia, vol. 2, pp. 420 ff., Leipzig: Teubner

Κατσιγιάννης Κώστας, (2010), «Από το Στομάχιον του Αρχιμήδη στο θεώρημα του Pick», Διπλωματική εργασία στο πλαίσιο του προγράμματος μεταπτυχιακών σπουδών ΕΚΠΑ- Πανεπιστημίου Κύπρου

Co-funded by the Erasmus+ Programme of the European Union